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Convective MHD Flow, concerning Non-Newtonian Fluid, through a porous medium, over a Porous Plate, in oscillating state, with Suction

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Abstract:

The paper studies and analyses the result of a convective flow, in a porous medium, concerning viscous Non- Newtonian fluid, electrically conducting, past a porous plate of semi-infinite nature in oscillating state influenced by transverse magnetic field. Values of temperature, velocity, heat flux and Skin Friction have been derived by solving the problem analytically. Effects of magnetic field, permeability, heat source, suction parameters, Prandtl and Hartmann numbers have been obtained using graphs and tables.

Key words: *Non- Newtonian, MHD convective flow, Porous Medium, Heat flux, Suction.*

Introduction:

The convective MHD flow has become a centre of attraction for many researchers and acquired a great importance due to its vide range applications in several fields. It plays vital role in chemical engineering, aerospace technology, food processing, , polymer technology, paper industry, wire and fibre coating, heating and cooling chambers, purification of molten metals, solar power technology and many other technological fields like nuclear power plants, satellites and missiles as well.

Dass, S.S. et.al. 2008 examined mass transfer impact on MHD flow of free convection pertaining to viscous fluid, limited within a porous oscillating plate in the presence of heat sourcein slip flow regime. Veera Krishna et.al. Analysed Convective transfer of both heat and mass on Peristaltic Williamson MHD fluid flow influenced by inclined magnetic field. Raghunath K.R et.al.Probed into Hall effects on free convective MHD flow in rotating state, with in porous medium over infinite vertical plate. Siva Prasad et.al. Studied the impact of radiation absorption and inclined magnetic field on a flow pertaining to mixed convection of radiation and chemically reacting fluid over a porous plate, infinite in nature. Gangadhar. R. et. .al, .Examined free convective MHD rotating flow of fluid, viscoelastic in nature, over an oscillating infinite, vertical plate of porous nature. Raghunath. K..R. et.al.Studied heat and mass transfer impactover an infinite oscillating vertical plate of porous nature pertaining to viscous MHD fluid flow.

2. Analytical solution of the problem:

i) Mathematical interpretation of Flow:

Impact of convective flow, in porous medium, concerning viscous Non- Newtonian fluid, electrically conducting, past a porous plate of semi-infinite nature in oscillating state influenced by transverse magnetic fieldB_0has been studied. In figure -1,velocity u is taken along x axis and velocity v along the y axis, which are normal to the plate. Considering involved physical quantities to be functions of y and t, Induced magnetic field being negligible and Reynolds number very small, the pressure is considered constant. Suction velocity at the plate taken as V_0. The equation of continuity shall be

$$\frac{\partial v}{\partial y}=0 \quad \ldots\ldots (1)$$

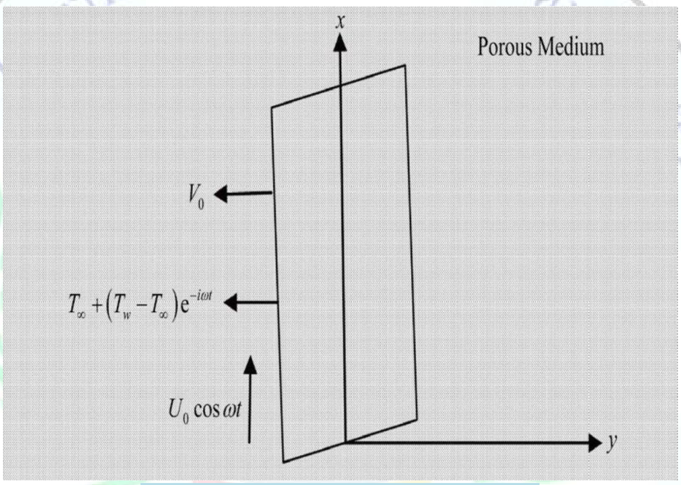

Figure 1. Physical model of the problem.

Considering y=0, v=$-V_0$ boundary layer eq. will be

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = (v+\alpha\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) - \frac{V}{K_0}u - \frac{\sigma {B_0}^2 u}{\rho} \quad \ldots\ldots\ldots(2)$$

$$\frac{\partial T}{\partial t} - V_0\frac{\partial T}{\partial y} = k[1+\alpha\frac{\partial}{\partial t}]\frac{\partial^2 T}{\partial y^2} - S[T-T_\infty] - \frac{\partial q_r}{\partial y} \quad \ldots\ldots\ldots .(3)$$

The allied conditions regarding boundary are

$$y=0:\ u=U_0\ coswt,\ \ T=T_\infty+(T_\omega-T_\infty)\,e^{-i\omega t}\ ]$$

$$y\rightarrow\infty:\ u\rightarrow 0,\ \ T\rightarrow T_\infty] \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

here k is thermal diffusivity, α is stressmoduli,u the flow velocity component in the x-direction,g is gravitational force, porous medium permeabilityK_0, density ρ, v kinematic viscosity, β stands for volumetric heat transfer expansion coefficient, S heat source parameter, T_∞ stands for fluid temperature remote from the plate, T for temperature.

According to Rosseland approximation related to radiative heat flux [Brewster (1992)]

$q_r = -\frac{4\sigma}{3\delta}\frac{\partial T^4}{\partial y}$ Here δ means coefficient of absorption, σ stands for Stefan – Boltzmann constant.

Since differences in temperaturewithin the flow are appreciably small so T^4 may be expressed as linear function of the temperature.

By Taylor series expansion, T^4 can be written as,

$T^4 \approx 4TT_\infty^3 - 3T_\infty^4$, higher powers of T having neglected.

ii) Evolving the dimensionless quantities

$t' = {U_0}^2 \frac{t}{v}$,$V_0' = \frac{V_0}{u_0}$,$\acute{\omega} = \frac{v\omega}{{U_0}^2}$

$y' = U_0 \frac{y}{v}$, $\acute{u} = \frac{u}{U_0}$, $\acute{S} = \frac{vS}{{U_0}^2}$, $\acute{\alpha} = \alpha \frac{{U_0}^2}{v}$

$\acute{T} = \frac{T - T_\infty}{T_\omega - T_\infty}$, $M = \frac{B_0}{U_0}(\frac{v\sigma}{\rho})$, $P_r = \frac{v\rho C_p}{k}$

$G_r = vg\beta \frac{T_\omega - T_\infty}{{U_0}^3}$, $K_p = \frac{K_0 {U_0}^2}{V^2}$,$R = \frac{16\sigma T_\infty^3}{3\delta k}$

Here P_r, G_r , M , stand for Prandtl number, Grashof number and Hartman numbers respectively,q_r for Radiative heat flux. Equations (2), (3) and allied boundary condition (4) reduce to,

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = [1 + \alpha \frac{\partial}{\partial t}] \frac{\partial^2 u}{\partial y^2} + G_r T - [\frac{1}{K_p} + M^2] u \quad \text{.............(5)}$$

$$\frac{\partial T}{\partial t} - V_0 \frac{\partial T}{\partial y} = \frac{1}{P_r} [1 + \alpha \frac{\partial}{\partial t}] \frac{\partial^2 T}{\partial y^2} - ST + \frac{R}{P_r} \frac{\partial^2 T}{\partial y^2} \quad \text{............ (6)}$$

(Dropping sign ′for convenience)

The conforming conditions of boundary are

$u = \cos wt$, $T = e^{-i\omega t}$ at y = 0 .... (7)

$u \to 0$, $T \to T_\infty$ as $y \to \infty$

Solving the equations:

Dropping ′ and solving non-linear partial differential equation (5), (6) including boundary condition (7),

$$u = u_o e^{i\omega t} + u_1 e^{-i\omega t} \quad \text{............ (8)}$$

$$T = T_0 e^{i\omega t} + T_1 e^{-i\omega t} \quad \text{............ (9)}$$

Whereu_i , T_i (i = 0 ,1) are functions of y only and $u_0 u_1$, T_0, T_1 are unknown, to be determined.

Equating Harmonic and non-Harmonic terms andPutting the values of u and T in equations (5) and (6) from equations (8), (9)

Obtained Ordinary differential equations are given as:-

$$\frac{d^2 u_0}{dy^2} + \frac{V_0}{1 - \alpha i\omega} \frac{du_0}{dy} - \frac{[\frac{1}{K_p} + M^2 + i\omega \;]}{1 - \alpha I\omega} u_0 = \frac{-G_r T_0}{1 - \alpha i\omega} \quad \text{............ (10)}$$

$$\frac{d^2u_1}{dy^2}+\frac{V_0}{1-\alpha i\omega}\frac{du_1}{dy}-\frac{[i\omega-\frac{1}{Kp}+M^2]}{1-\alpha i\omega}u_1=\frac{-G_rT_1}{1-\alpha i\omega} \quad \ldots\ldots\ldots(11)$$

$$\frac{d^2T_0}{dy^2}+\frac{V_0P_r}{1+R+\alpha i\omega}\frac{dT_0}{dy}-\frac{P_r(S+i\omega)}{1+R+\alpha i\omega}T_0=0 \quad \ldots\ldots\ldots(12)$$

$$\frac{d^2T_1}{dy^2}+\frac{V_0P_r}{1+R-\alpha i\omega}\frac{dT_1}{dy}-\frac{(S-i\omega)}{1+R-\alpha i\omega}T_1=0 \quad \ldots\ (13)$$

The resultantboundary conditions are,

at y = 0 $u_0=\frac{1}{2}u_1=\frac{1}{2}$......... (14)

at Y→∞$u_0=0$ $u_1=1$.......... (15)

at y = 0 $T_0=0$ $T_1=1$.......... (16)

at Y→∞$T_0=0$ $T_1=0$......... (17)

From equation (12) and (13), obtained ordinary differential equations (16) and (17) with prescribed boundary conditions which are solved as under:

T_0= 0 ...... (18)

T_1= exp ($-m_4\,y$) .... (19)

Where $m_4=\frac{P+\sqrt{P^2+4Q}}{2}$ where $P=\frac{V_0P_r}{1+R-\alpha i\omega}$, $Q=\frac{P_r(S-i\omega)}{1+R-\alpha i\omega}$

Now using the equations (18) and (19) in equation (10) and (11) we get differential equations of second order.

$u_o$and$u_1$are given by :

$$\frac{d^2u_0}{dy^2}+\frac{V_0}{1+\alpha I\omega}\frac{du_0}{dy}-\frac{\frac{1}{kp}+M^2+I\omega}{1+\alpha i\omega}u_0=0 \quad \ldots\ldots\ .(20)$$

$$\frac{d^2u_1}{dy^2}+\frac{V_0}{1-\alpha i\omega}\frac{du_1}{dy}-\frac{i\omega-\frac{1}{Kp}-M^2}{1-\alpha i\omega}u_1=\frac{-G_r\exp(-m_4y)}{1-\alpha i\omega} \quad \ldots\ldots\ .(21)$$

Using the boundary conditions of$u_o$and u_1 equations (20) and (21) are known and given by

$$u_0=\frac{1}{2}\exp(-m_6y) \quad \ldots\ldots\ .(22)$$

$$u_1=\left(\frac{1}{2}-C_5\right)\exp(-m_8y)+C_5\exp(-m_4y)\ldots\ldots(23)$$

Where $m_6=\frac{A+\sqrt{A^2+4B}}{2}$

Where $A=\frac{V_0}{1+\alpha i\omega}$, $B=\frac{\frac{1}{kp}+M^2+i\omega}{1+\alpha i\omega}$ $m_8=\frac{C+\sqrt{C^2+4E}}{2}$

$C=\frac{V_0}{1-\alpha i\omega}$, $E=\frac{\frac{1}{kp}+M^2-i\omega}{1-\alpha i\omega}$

Now substituting the values of $u_o$and u_1, T_0 and T_1 in equation (8) and (9)

$$u=\frac{1}{2}(\exp(-m_6y))e^{i\omega t}+\left\{\left(\frac{1}{2}-C_5\right)(\exp(-m_8y))+C_5(\exp(-m_4y))\right\}e^{-i\omega t}\ \ldots(24)$$

Where$C_5=\frac{1}{m_4^2-Cm_4-E}\left(\frac{-G_r}{1-\alpha I\omega}\right)$

T = $\exp(-m_4 y)\, e^{-i\omega t}$……….. (25)

Results Interpretation:

Present study involves Impact of convective flow, in a porous medium, concerning viscous Non- Newtonian fluid, electrically conducting, past a porous plate of semi-infinite nature in oscillating state influenced by transverse magnetic field. Mathematical expressions pertaining to temperature and velocity fields have been derived analytically solving the field equations. figures 2-5show the effect of parameters S, R, Pr, v0 on temperature field T and figure 6-12 of flow parameters Pr, Kp, R, M,

Gr, v0 and S on

Velocity field u.

Figure -2, It shows that taking varying values of S, heat source parameter, temperature T is plotted against y. It means that the temperature field T decreases sharply near the boundary when the value of y increases and approaches zero at the boundary.

Figure -3, It shows that taking different values of radiative parameter R temperatureT is plotted against y. Increase in Radiative parameter R results in decrease of temperature which finally approaches zero at the boundary.In Figure- 4,Taking different values of Pr, Prandlt number plotting of temperature T is affected. It came to fore that value of Tdecreases and finally approaches zero at the boundary. Figure- 5, in this figure it is shown that taking different values ofv0, suction velocity, Temperature is plotted against y, There is sharp decrease in temperature when y increases and finally approaches zero at the boundary.It appears from the figure- 6 that velocity field isplotted taking varying values of Prandlt number. It is seen that when y increases velocity field decreases and finally approaches the boundary. Figure-7, Itappearsthatfor different values of Permeability parameter velocity field is shown against y. It is inferred thatincrease in value of y brings Sharp decrease in velocity field which finally reaches zero at the boundary. Figure-8, figure shows that Velocity field is plotted taking varying values of Radiative parameter. Increase in y results inSharp decrease in the velocity filed which finally reaches zero at the boundary. Figure-9, it appears from the figure that velocity field is plotted with varying values of magnetic field. It infers that due to retardation in fluid flow due to increase in magnetic field velocity field decreases and finally approaches zero at the boundary. Figure-10, in this figure velocity filed is shown plotted taking different values of thermal buoyancy force parameter Gr. velocity decreases due to increase in values of Gr and finally reaches zero at the boundary. Figure-11, in this figure, taking different values ofsuction velocity v0velocity field is shown plotted.When value of y increases there is decrease in velocity field which finally approaches zero at the boundary. Figure-12. In this figure velocity filed is shown plotted with varying values of heat source parameter S. It gives rise to Sharpe decrease in velocity forincrease in the value of S, which finally approaches zero at the boundary.

Table-1. It contains the value of $C_f$the Skin-friction. Due to heat source parameter S increasesk in-friction decreases. It also decreases whenPrandltnumber and magnetic parameter increase. But increase in Gr, Radiation and permeability parameters, skin-frictiongets increase.

Table-2. RepresentsNusseltnumber Nu. Increase in heat source parameter S decreases Nusselt number. It also decreases when prandlt number Increases. Value of Nusselt number increases when there is increase in Radiation parameter.

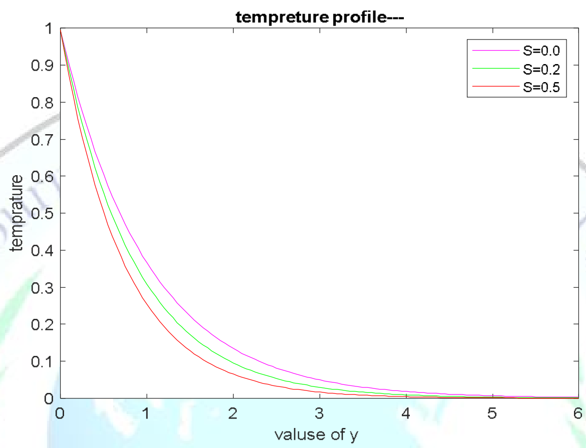

Figure-2. at R=2, alpha=0.3, Pr=3, w=0.2,exp=2.7183,t=0.5,v0=1. Temperature varies with heat source parameter

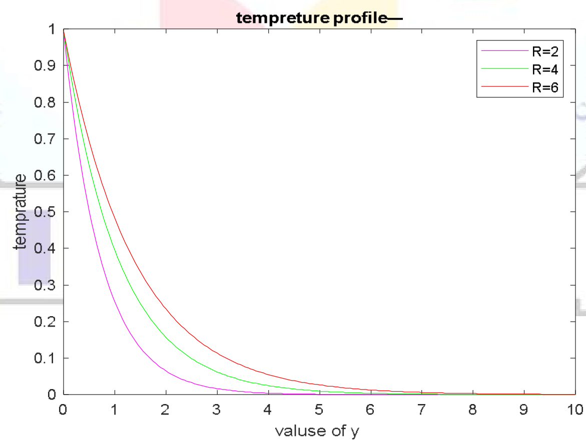

Figure-3. at S=0.5, alpha=0.3, Pr=3, w=0.2, exp=2.7183,t=0.5,v0=1. Radiation parameter affects variation in temperature.

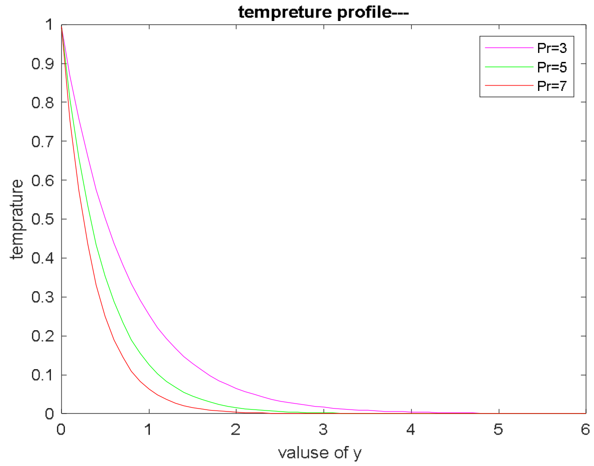

Figure-4. at S=0.5, alpha=0.3, R=2, w=0.2, exp=2.7183, t=0.5, v0=1.
Prandtl numberbrings about variation in temperature

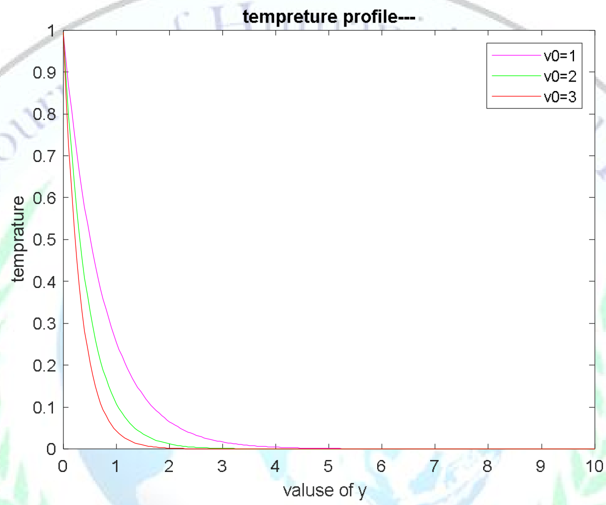

Figure-5. at S=0.5, alpha=0.3, R=2, w=0.2, exp=2.7183, t=0.5, Pr=3.
The temperature varies with suction velocity

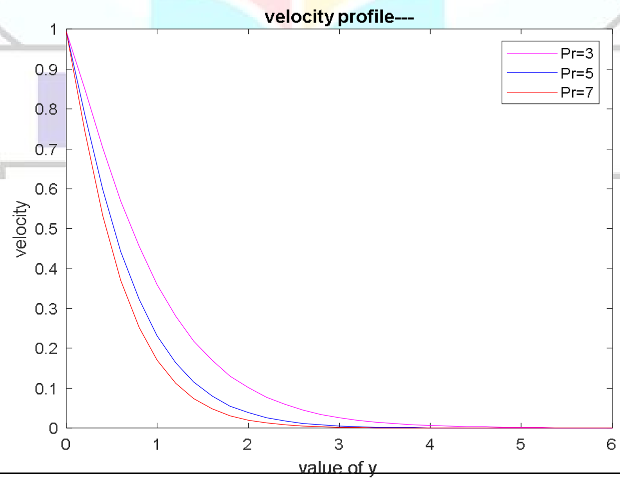

Figure-6.at S=0.5, alpha=0.3, R=2, w=0.2, exp=2.7183, t=0.5, v0=1,M=0.5,Kp=0.3,Gr=5.
With Prandtl number velocity varies

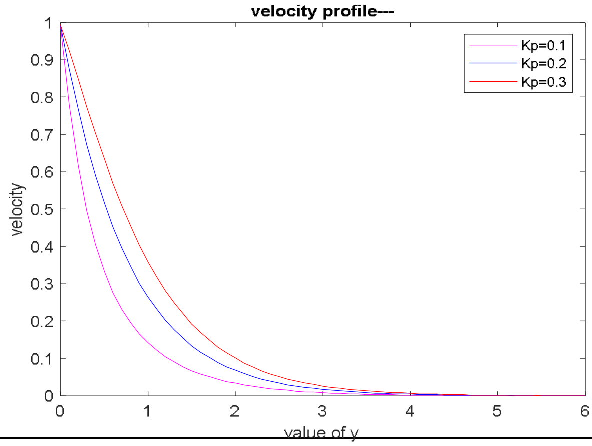

Figure-7. at S=0.5, alpha=0.3, R=2, w=0.2, exp=2.7183, t=0.5, v0=1,M=0.5,Pr=3,Gr=5. The velocity varies with Permeability

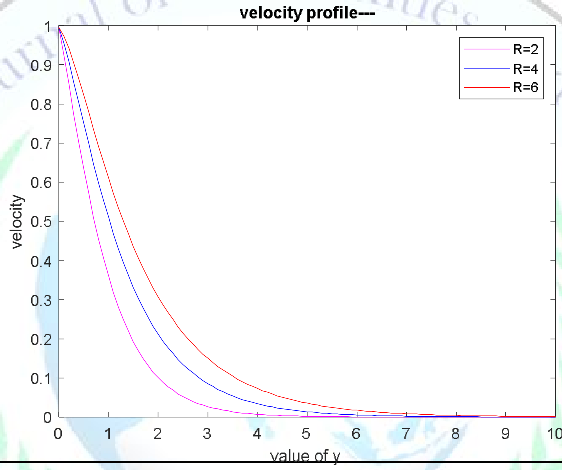

Figure-8.at S=0.5, alpha=0.3, w=0.2, exp=2.7183, t=0.5, v0=1,M=0.5,Pr=3,Kp=0.3,Gr=5. Radiation parameter brings variation in velocity

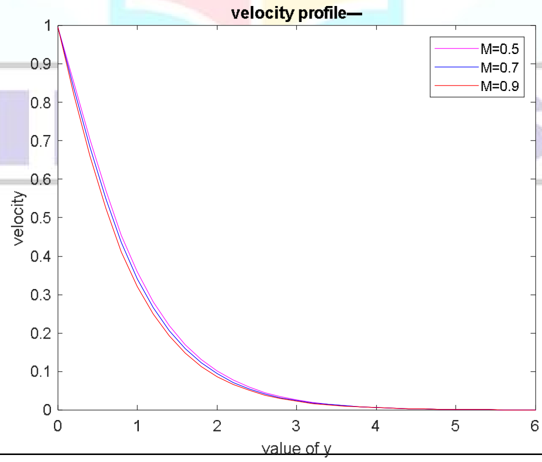

Figure-9. at S=0.5, alpha=0.3, w=0.2, exp=2.7183, t=0.5, v0=1,R=2,Pr=3,Kp=0.3,Gr=5.The velocity varies with Magnetic field parameter

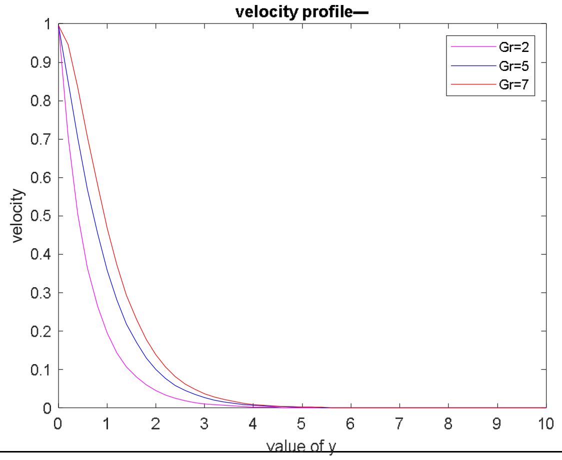

Figure-10. at S=0.5, alpha=0.3, w=0.2, exp=2.7183, t=0.5, v0=1,R=2,Pr=3,Kp=0.3,M=0.5. Grashof numbercauses variation in velocity

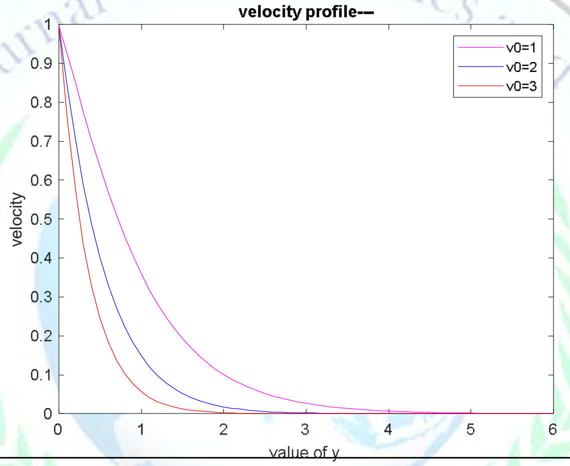

Figure-11.at S=0.5, alpha=0.3, w=0.2, exp=2.7183, t=0.5, Gr=5,R=2,Pr=3,Kp=0.3,M=0.5.With suction velocity Velocity varies

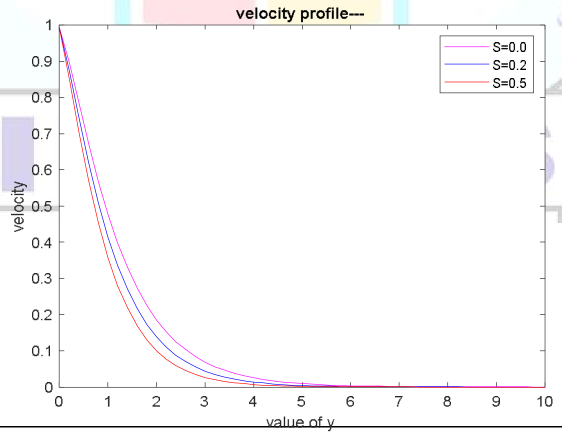

Figure-12. at v0=1, alpha=0.3, w=0.2, exp=2.7183, t=0.5, Gr=5,R=2,Pr=3,Kp=0.3,M=0.5. thevelocity varies with heat source parameter .

Skin Friction (C_f) at the plate

It is given by:

$$C_f = \left[\frac{\partial u}{\partial y}\right]_{y=0} = \frac{1}{2}(-m_6)e^{i\omega t} + \left\{\left(\frac{1}{2} - C_5\right)(-m_8) + C_5(-m_4)\right\}e^{-i\omega t} \quad \text{........(26)}$$

Table 1. Showing computed values of C_f

S	Gr	M	Pr	R	Kp	C_f
0.0	5	0.5	3	2	0.3	-0.4210
0.2	5	0.5	3	2	0.3	-0.5524
0.5	5	0.5	3	2	0.3	-0.6831
0.5	2	0.5	3	2	0.3	-1.7377
0.5	7	0.5	3	2	0.3	0.0201
0.5	5	0.7	3	2	0.3	-0.7798
0.5	5	0.9	3	2	0.3	-0.9023
0.5	5	0.5	5	2	0.3	-1.0338
0.5	5	0.5	7	2	0.3	-1.2633
0.5	5	0.5	3	4	0.3	-0.3572
0.5	5	0.5	3	6	0.3	-0.1660
0.5	5	0.5	3	2	0.5	-0.0383
0.5	5	0.5	3	2	0.7	-0.3259

Coefficient of heat transfer

It is given by:

$$Nu = \left[\frac{\partial T}{\partial y}\right]_{y=0} = (-m_4)e^{-i\omega t} \quad (27)$$

Table 2. Showing computed value of Nu

S	**Pr**	**R**	**Nu**
0.0	3	2	-0.9946
0.5	3	2	-1.3588
0.2	3	2	-1.1646
0.5	5	2	-2.0584
0.5	7	2	-2.7419
0.5	3	4	-0.9198
0.5	3	6	-0.7207

Conclusion:

- When the value of Grashof number Gr increases, there is increase in Permeability parameter, Kp but Radiation parameter R increases the value of skin-friction.
- Value of skin-friction decreases for Increase in Prandlt number, magnetic field and heat source parameter.
- Due to increase inPrandlt number Pr and heat source parameter S, decreases Nusselt number.
- Nusselt number increases by the increase of Radiation parameter, R.
- Increase in S, heat source parameter , Gr, Grashof number, Magnetic parameter M, Pr,Prandlt number, Permeability parameter , Kp and Radiation parameter R decrease both velocity profile u and temperature profile T.

References:

[1] Manna, S. S., Das, S. and Jana, R.N. 2012. Effects of radiation on unsteady MHD free convective flow past an oscillating vertical porous plate embedded in a porous medium with oscillatory heat flux. Advances in Applied Science Research, 3, 3722-3736.

[2] Shen, F., Tan, W., Zhao, Y. and Masuoka, T. 2006. The Rayleigh-Stokes problem for a heated generalized second grade fluid with fractional derivative model. Nonlinear Analysis: Real World Applications, 7,1072-1080**.**

[3] Singh, P. and Gupta, C.B. 2005. MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Ind. J. Theo. Phys., 53, 111-120.

[4] Choudhury, R. and Das, S.K. 2014. Visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer. Journal of Applied Fluid Mechanics, 7, 603609.

[5] Deka, R.K., Das, U.N. and Soundalgekar, V.M. 1997. Free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux. Ind. J. Math, 39, 195-202.

[6] Das, S.S., Tripathy, R.K., Sahoo, S.K. and Dash, B.K. 2008. Mass transfer effects on free convective MHD flow of a viscous fluid bounded by an oscillating porous plate in the slip flow regime with heat source. J. Ultra Scientist of Phys. Sci. 20, 169-176.

[7] Hayat, T., Mohyuddin, M.R., Asghar, S., Siddiqui, A.M. (2004), The flow of a visco-elastic fluid on an oscillating plate. Z. Angew. Math. Mech., 84, 65-70.

[8] VeeraKrishna.M and B.V.Swarnalathamma, Convective Heat and Mass Transfer on MHD Peristaltic Flow of Williamson Fluid with the Effect of Inclined Magnetic Field," AIP Conference Proceedings, vol. 1728, p. 020461, 2016. DOI: 10.1063/1.4946512

[9] Swarnalathamma. B. V. and M. Veera Krishna, Peristaltic hemodynamic flow of couple stress fluid through a porous medium under the influence of magnetic field with slip effect AIP Conference Proceedings, vol. 1728, p. 020603, 2016. DOI: 10.1063/1.4946654

[10] VeeraKrishna.M and M.Gangadhar Reddy MHD free convective rotating flow of Visco-elastic fluid past an infinite vertical oscillating porous plate with chemical reaction, IOP Conf. Series: Materials Science and Engineering, vol. 149, p. 012217, 2016 DOI: 10.1088/1757-899X/149/1/012217.

[11] Obulesu M, Siva Prasad R;,Effect of Inclined Magnetic Field and Radiation Absorption on Mixed Convection Flow of a Chemically Reacting and Radiating Fluid Past a Semi Infinite Porous Plate, .i-Manager's Journal on Mathematics; Nagercoil Vol. 7, Iss. 4, (Oct/Dec 2018): 39-49. DOI:10.26634/jmat.7.4.1556.

[12] Raghunath K, R Sivaprasad, GSS Raju, Hall Effects on MHD Convective Rotating Flow of Through a Porous Medium past Infinite Vertical Plate Annals of Pure and Applied Mathematics Vol. 16, No. 2, 2018, 353-263 ISSN: 2279-087X (P), 2279-0888(online), DOI: http://dx.doi.org/10.22457/apam.v16n2a1.

[13] Raghunath K, M.V Krishna, R Sivaprasad, GSS Raju, Heat and Mass Transfer on MHD Flow of a Visco-Elastic fluid past an infinite verticle oscillating Porous plate, British Journal of Mathematics, ISSN: 2231-0851,Vol.: 17, Issue.: 6, DOI : 10.9734/BJMCS/2016/25872