INTERNATIONAL RESEARCH JOURNAL OF HUMANITIES AND INTERDISCIPLINARY STUDIES
(Peer-reviewed, Refereed, Indexed & Open Access Journal)
DOI : 03.2021-11278686 ISSN : 2582-8568 IMPACT FACTOR : 8.428 (SJIF 2026)

Contribution of Ancient Indian Mathematicians to Number Theory and Algebra

Rushikesh Vilas Lahane

Assistance Professor (CHB)
Department of Mathematics,
Shri Muktanand College, Gangapur.
Affiliated: Dr. Babasaheb Ambedkar Marathvada University,
Chh. Sambhajinagar (Maharashtra, India)
E-mail: rushikeshvlahane@gmail.com

DOI No. 03.2021-11278686 DOI Link :: https://doi-ds.org/doilink/08.2026-37534699/IRJHIS2608019

Abstract:

Ancient Indian mathematics played important roles in the development of concepts and methods that are now incorporated into number theory and algebra, as well as in notation. This paper investigates the evolution of place value notation in decimal systems, the use of zero, negative numbers, indeterminate equations, quadratic equations and cyclic methods for solving Pell-type equations. It is based on historical and analytical approach with emphasis on the works of Aryabhata, Brahmagupta, Mahavira and Bhaskara II; and on the overall mathematical tradition in Sanskrit of which they worked. The study suggests that; besides the significance of the discoveries, the significance of Indian Mathematics also includes its algorithmic style: the rules were formulated as general procedures, exemplified by a few cases, and communicated in commentaries. These accomplishments reinforced the computational aspects of arithmetic, broadened the range of numbers, and developed advanced techniques for equations whose solutions are limited to integers. Indian numerical and algebraic knowledge also had an impact on Islamic mathematics and European mathematical development, through translation and subsequent scholarly exchange.

Keywords*: Indian mathematics, number theory, algebra, zero, Aryabhata, Brahmagupta and Bhaskara II.*

Introduction:

Traditional history of mathematics was told primarily in the Greek, Islamic and European contexts. But more recent scholarship shows that mathematical knowledge developed because of interaction among multiple civilizations, and that India was one of the major centers of this development (Joseph, 2011; Plofker, 2009). In the early centuries of the Common Era to the 12th century, Indian scholars developed a strong tradition of computational practice that was tightly coupled to the subjects of arithmetic, algebra, geometry and astronomy. While Bhaskara II is generally considered a medieval mathematician, this paper adopts the broad perspective in using the term "ancient Indian mathematics" to cover the continuum of classical and early medieval Sanskrit mathematics, including his work.

The conceptual and procedural contribution of the Indians to number theory and algebra. The Indian scholars formulated decimal place value numeration and invented arithmetical concepts involving zero and negative numbers, solved linear equations and quadratic equations, and explored equations that had integer solutions. They were often expressed in short Sanskrit stanzas with examples and commentaries. This format promoted memorization but also maintained general algorithms that could be used in numerous problems (Plofker, 2009). The main thesis of this paper is that it was the ancient Indian mathematicians who revolutionized mathematics by introducing an expanded domain of number and systematic methods of computation and equation solving.

Method and Scope:

This paper is based on the historical-analytical method. It is based upon the translations of primary Sanskrit texts, particularly the Aryabhatiya, and the algebraic works of Bhaskara II and Brahmagupta, along with modern history of Indian mathematics (Clark, 1930; Colebrooke, 1817; Datta & Singh, 1935, 1938; Plofker, 2009). The discussion is limited to contributions that are directly related to number theory and algebra and not astronomy, geometry, or trigonometry. Special emphasis is placed on claims that are traceable to extant texts, and not on the notion that each concept of Indian mathematics was "invented" by one person.

Decimal place value, Numerals and Zero.

The Indian decimal system of place value was the essential basis of Indian arithmetic. A place-value system is one in which the same symbol represents a unit, ten, one hundred, or any power of 10, depending on its position. This permits very large numbers to be represented in a compact manner and provides a more efficient way of making calculations in writing than systems using repeated additive symbols. In Indian sources, the evolution from number words and numeral signs to a full-fledged decimal positional system (Datta & Singh, 1935; Plofker, 2009) is not sudden but gradual.

Zero was important because if one of the powers of ten was missing, that had to be indicated by an empty space. But the Indian success extended beyond the mere fact of their using zero as a place holder. In 628 CE, Brahmagupta wrote his Brahmasphutasiddhanta, which established zero as the difference between two identical numbers and provided rules for computation with zero, positive numbers and negative numbers (Colebrooke, 1817; Plofker, 2009). He correctly said that adding or subtracting zero does not change a number and that a number multiplied by zero is zero. By modern standards, his rule for division by zero was not mathematically sound, but historically important because he was not simply dealing with empty space, but with zero as an object of arithmetic.

Positive and negative numbers were represented by the terms "fortune" and "debt" respectively, also enlarging the number system. It is a commercial language that puts abstract sign rules into perspective: a debt taken from nothing is a fortune, and a debt multiplied by a fortune is a debt. These rules allowed equations with negative numbers to be manipulated. Indian mathematicians included negative numbers in the general procedure of arithmetic and algebra in a clear way, while negative numbers were introduced in other civilisations. (Joseph, 2011)

The algorithms of which the numeral system formed a part were as closely woven into the strength of the system as the system itself. Multiplication and division, root extraction, series, and checking procedures were described in Sanskrit using ten forms that used powers. A few texts used word or syllable notation to enable the use of numerical rules in metrical verse, and a few calculations could still rely on the positional notation. The exchange of knowledge between the oral, the written, and the computational facilitated the circulation of mathematical knowledge among teachers, students, astronomers, and commentators. So, India did not just provide 10 numbers. It was an integrated culture of representation and calculation which made more and more sophisticated algebra manageable (Datta & Singh, 1935, Plofker, 2009).

The Kuttaka method and Aryabhata.

The Aryabhatiya by Aryabhata is one of the earliest Indian classical texts in existence, dated 499 CE. It has a mathematical section with algorithms for taking square and cube roots, for solving practical problems and for indeterminate equations (Clark, 1930). Aryabhata's most important contribution to number theory was the kuttaka, or "pulverizer," a method for finding integer solutions to linear indeterminate equations.

A linear indeterminate equation is one that can be written in the form -

$ax + c = by$

Does not request one solution to a relation—one real-number solution. Aryabhata's procedure is repeated division with remainders like continued fractions and the Euclidean algorithm. The numbers were metaphorically "pulverized" to smaller sizes until a workable relation was obtained. The process may be able to solve problems such as astronomical cycles, remainders, and simultaneous congruence-like conditions.

The kuttaka is significant in number theory because it emphasizes divisibility and integer solutions. It is prepared for systematic study of linear Diophantine equations and congruences. However, Aryabhata did not employ modern symbolic notation, and his verbal rule was algorithmic: Given the coefficients, a finite number of operations yielded the solutions. Mathematicians later, such as Brahmagupta, Mahavira and Bhaskara II, developed and extended the method (Datta & Singh, 1938).

An algebra and number theory problem book by Square-Upside-Down

Brahmagupta was one of the most systematic formulators of ancient Indian algebra. Arithmetic is dealt with directly in two chapters of the Brahmasphutasiddhanta, and algebra, such as operations with signed numbers, fractions, surds, quadratic equations and indeterminate equations. He could use an equation as a computational problem, a problem with general rules rather than a set of discrete numerical puzzles (Colebrooke, 1817).

For quadratic equations, Brahmagupta gave algorithms equivalent to variants of the present-day quadratic formula. He would allow negative values in intermediate calculations, but Indian writers might assign negative answers to problems based on the context of the problem. He also had rules for working with unknowns and for simplifying expressions with roots in his algebra. This was a significant step towards abstract algebraic thinking, even if it was still rhetorical and not symbolic.

Brahmagupta's most notable number-theoretic work was with equations written as

$Nx^2 + k = y^2$.

He discovered a composition principle showing how two solutions of equations of this type could be combined to create another solution. In modern notation, the identity

$(a^2 - Nb^2)(c^2 - Nd^2) = (ac + Nbd)^2 - N(ad + bc)^2$

shows why such composition works. The principle, later called bhavana, became essential to the Indian treatment of equations of the form

$x^2 - Ny^2 = 1$.

The equations are frequently referred to as Pell's equations but the investigation by an Indian occurred many hundreds of years before the English mathematician John Pell. Brahmagupta showed that if he could make up a small value of k, he could use composition to find a solution for k = 1. This is a study at a very deep level, as evidenced by his famous challenge with N = 92. The method did not result in the most efficient general procedure but had laid the groundwork for the later cyclic methods (Datta & Singh, 1938; Selenius, 1975).

The association between Mahavira and the Organization of Algebra.

Mahavira, a Jain mathematician, produced a broad mathematical work, the Ganita-sara-sangraha, in the ninth century, before an astronomical work. The organization of the book reflects the increasing importance of mathematics as a separate subject. While providing detailed classifications of computational problems (Plofker, 2009), Mahavira talked about fractions, ratios, permutations, combinations, areas, volumes and equations.

In algebra he explained the rules for linear and quadratic equations and used the kuttaka method in integer-solution problems, in number theory. His examples helped to make people understand the general procedures. Mahavira also modified some of the rules previously set. For instance, he did not believe that a negative number could have a square root that was a real number, in keeping with the real-number system in use at the time. What he did to this was not so much a new revolutionary theorem as a systematic synthesis and the teaching of the previous work of the Indians in algebra.

It is possible to write bhavanam and chakravala using Bhaskara II.

The classical Sanskrit algebraic tradition was carried to a high level in the 12th century by Bhaskara II. The Lilavati on arithmetic and the Bijaganita on algebra were part of his Siddhanta Shiromani. He carried out further research on zero, negative numbers, surds, equations and indeterminate analysis. In solving equations, Bhaskara noticed that a positive square has two square roots, which was an important step towards the serious treatment of multiple solutions to algebraic equations (Colebrooke, 1817; Plofker, 2009).

The chakravala, or "cyclic," method for equations of the form $x^2 = n$ was his greatest contribution to number theory, presented and developed in this manner.

$x^2 - Ny^2 = 1$.

It is not possible to say that Bhaskara was the only creator of the method since it was probably developed much earlier, especially by Jayadeva. However, his presentation made it popular and proved effective (Selenius, 1975).

Chakravala starts with a triple satisfying.

$x^2 - Ny^2 = k$

and then repeatedly replaces it with a new triple with a small value of $|k|$. The process will often finish with $k = 1$ in a very efficient manner, through cyclic compositions and a clever selection of an extra integer. Bhaskara was able to use the method in tough cases such as $N = 61$, for which the smallest positive solution is very large. The success of the chakravala is remarkable, as it is not a random selection of methods: approximation, divisibility, algebraic composition and optimization are all used and repeated in an algorithm.

Later, the European mathematicians studied the same class of equations, but with continued fractions. Mathematically similar, but different, in spirit and procedure, is the Indian cyclic method. It is a particularly sophisticated example, showing that Indian number theory during the ancient and medieval period was not just a collection of numerical solutions, but also of powerful general methods for solving general nonlinear integer equations.

Wavelength and Global Significance

Indian mathematics did not stay within its geographical limits. Since the eighth century, Sanskrit astronomical and mathematical texts were translated or adapted in the Islamic world. Arithmetic and the system of counting in India (using Indian numerals) and the use of decimal place values entered Arabic mathematical culture. Al-Khwarizmi and other scholars spread Indian numerals for calculation; these techniques spread into Latin Europe (Joseph, 2011; Plofker, 2009).

Transmission should not be thought of as a single unidirectional movement. The Indian and Islamic scholars, the Greeks, and subsequently the Europeans, chose, translated, criticized, and adapted the ideas of each other. However, the Indian legacy of decimal positional numeration is a direct and lasting impact on the world. The algorithmic orientation of Indian texts is also a crucial element. The kuttaka, bhavana and chakravala reveal a consistent fascination with finite procedures which would solve general classes of problems.

They also make the story of the development of algebra less simple in terms of symbolic notation. Indian algebra was mainly rhetorical, but also general, abstract, and powerful in its use of calculation. Symbols are very useful but as the Indian example shows, important algebraic breakthroughs can be achieved by means of a carefully formulated verbal rule and a few numerical examples.

History in and as interpretation, and the limits of attribution.

Care is needed in assessing these achievements. The Sanskrit scholarship did not have the modern categories of "number theory", "algebra" and "Pell equation" in their present form. The various problems were mostly related to astronomy, calendrical calculations, commerce, measurement, or pastimes in mathematics. The historians thus translate ancient procedures into modern notation to explain them, but differences in purpose and reasoning can be obscured by the modern notation. To illustrate its efficiency – perhaps even lend the name continued-fraction algorithm to it – it might be called an early continued-fraction algorithm; but it is not a copy of the Indian method of continued fractions, with its own unique choices and vocabulary (Plofker, 2009; Selenius, 1975).

The attribution should also continue to be cumulative. The chakravala was not one invention but rather an evolution of Zero. It is important to note that surviving texts are only a part of an even larger manuscript and teaching tradition, and that the date of a surviving copy may not be the date of the ideas contained in that copy. Thus, the notion of the invention of zero, algebra or any of the methods of indeterminate analysis by one man is historically uncorrect.

All these warnings do not diminish the significance of Indian mathematics. Instead, they show a lasting community of intellect, where achievements were passed down, judged, abstracted and imparted. The balance of history is to avoid being Eurocentric and to not overestimate the civilizational nature of history. It acknowledges the mathematical content of their texts and evidence of their influence on other mathematicians and knows that mathematical knowledge has always been acquired through exchange.

Conclusion:

Ancient Indian mathematicians made great strides in number theory and algebra by broadening the definition of the number and the scope of the equations they were able to solve. Place value of the decimal and zero made a convenient language of numbers. The algebraic rules for the negative quantities made algebraic operations more general. Aryabhata introduced an algorithm for linear integer equations; Brahmagupta invented signed arithmetic, quadratic procedures and constructed Pell-type equations; Bhaskara II developed a system of indeterminate analysis, called chakravala; and

Mahavira systematized and taught algorithms for algebra.

They did not achieve anything in isolation. Every scholar was assigned to a tradition of rules, examples, commentaries, and revisions. This tradition combined practical computation with abstract thought and eventually led to the transfer of mathematical knowledge from Asia to the Islamic world and Europe. Ancient India has, therefore, to be regarded as one of the major centres of creation of the history of number theory and algebra.

References:

1. Clark, W. E. (Trans.). (1930). *The Aryabhatiya of Aryabhata*. University of Chicago Press.
2. Colebrooke, H. T. (Trans.). (1817). *Algebra, with arithmetic and mensuration, from the Sanscrit of Brahmegupta and Bhascara*. John Murray.
3. Datta, B., & Singh, A. N. (1935). *History of Hindu mathematics: A source book* (Vol. 1, Numerical notation and arithmetic). Motilal Banarsidass.
4. Datta, B., & Singh, A. N. (1938). *History of Hindu mathematics: A source book* (Vol. 2, Algebra). Motilal Banarsidass.
5. Joseph, G. G. (2011). *The crest of the peacock: Non-European roots of mathematics* (3rd ed.). Princeton University Press.
6. Plofker, K. (2009). *Mathematics in India*. Princeton University Press.
7. Selenius, C.-O. (1975). Rationale of the chakravala process of Jayadeva and Bhaskara II. *Historia Mathematica, 2*(2), 167-184. https://doi.org/10.1016/0315-0860(75)90013-6